METEO 470 Climate Dynamics **Spring 2008**

PROBLEM SET #1

EQUATIONS OF MOTION AND ENERGY EQUATION

1. Consider a purely westerly (from the west) wind that is *horizontally uniform.* Assume that the effective eddy viscosity of the air ν is uniform, and that there are no zonal pressure gradients.

a. Write the equation of momentum conservation for this situation.

b. Find a *steady* solution U(z) that satisfies this equation? Assume a "no slip" boundary condition [U=0] on the velocity field at the lower boundary z=0, and assume that the wind at z=10m above the ground is U= 10m/s.

c. Find a *steady* solution U(z) that satisfies this equation as in "b" but assume, instead, a "free slip" boundary condition [$\partial U/\partial z=0$] on the velocity field at the lower boundary z=0.

d. Suppose that a simple "Rayleigh" friction law

$$\boldsymbol{f}=-k\boldsymbol{u}$$

(valid for u>0, where k is a positive constant) is used in place of the conventional "Newtonian" (eddy friction) law. Drive the steady solution U(z), and discuss how the solution compares to the solution obtained in (b) and (c).

2. Complete some steps that were left out in class:

a) In the derivation of the continuity equation, I stated that the result:

$$-\frac{1}{\rho}\frac{d\rho}{dt}=\nabla\cdot\mathbf{v}$$

follows from the result,

$$\frac{1}{\delta V}\frac{d(\delta V)}{dt}=-\frac{1}{\rho}\frac{d\rho}{dt}$$

Show the mathematical steps that were missing from lecture.

b) Use the chain rule to show:

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+\mathbf{v}\cdot\nabla\rho$$

c) Show how the combination of the continuity equation in the Lagrangian form

$$-\frac{1}{\rho}\frac{d\rho}{dt}=\nabla\cdot\mathbf{v}$$

with the above expansion of the material derivative

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+\mathbf{v}\cdot\nabla\rho$$

implies the Eulerian form,

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\mathbf{v})$$

3. We will consider a simple model for atmospheric temperatures. We will approximate the temperature field as zonally uniform, having the form,

$$T=T_0+T(\phi)exp(-z/H)$$

where T_0 **is a constant temperature,** $T(\phi)$ **is a (positive) function of latitude** ϕ **and** z **is the altitude above the surface, and** H **is a characteristic height scale. For simplicity, we will confine our consideration to the Northern Hemisphere. The velocity field is described by a constant positive (i.e. poleward) wind** $V=V_0$**. We neglect any possible radiative or latent heat flux in or out of the system, and any horizontal diffusion of heat, and assume that the pressure field is horizontally uniform and constant over time. Assume constant vertical diffusivity.**

a) Derive an equation that governs $T(\phi)$

b) What is the steady state solution of the temperature equation for $T(\phi)$. Discuss the two cases (1) $V_0>0$ (a *poleward* wind) and (2) $V_0<0$ (an *equatorward* wind). Do these solutions make sense physically? What important factors are missing?

4. Consider another model for atmospheric temperatures. We again approximate the temperature field as zonally uniform, having the form,

$$T=T_0+T(\phi)exp(-z/H)$$

where T_0 is a constant temperature, $T(\phi)$ is a function of latitude ϕ, z is the altitude above the surface, and H is a characteristic height scale. For simplicity, we will again confine our consideration to the Northern Hemisphere. We will ignore any vertical or horizontal diffusion of heat, and assume that the pressure field is horizontally uniform and constant over time. Unlike problem #3, we will assume no motion (i.e., all wind components are zero). Unlike problem #3, we will no longer neglect radiative and latent heating. We will assume that the total (longwave+shortwave) radiative and latent heating is given by

$$q= q_0cos(2\phi)$$

where q_0 is a positive constant.

a) Find the time-dependent solution for $T(\phi)$. Does this solution make sense physically? Is there a steady state solution? What key considerations are missing?