METEO 470 Climate Dynamics **Spring 2008**

PROBLEM SET #2

LARGE-SCALE ATMOSPHERIC CIRCULATION

1. Derive the Hadley Cell circulation on a *non-rotating earth*.

For simplicity, assume that the atmosphere density can be approximated by a *linear* equation of state

$$\rho = \rho_0[1 - \alpha(T - T_0)]$$

Assume the properties of the atmosphere to be zonally uniform, so that any circulation takes place in a latitude-vertical plane. We will proceed in several steps:

a. Make the 'Boussinesq' approximation (assume $\rho = \rho_0$ where appropriate). Assume that vertical eddy viscosities are zero, and assume a constant horizontal eddy viscosity ν. Neglect Earth's rotation, any non-linear effects, and assume a steady state. What are the expressions of vertical and meridional momentum balance for the large-scale circulation under these circumstances?

b. Combine the two equations from part 'a' into a single partial differential equation (containing both vertical and meridional derivatives). What is this equation?

c. Now, assume a temperature profile that is a function of latitude and altitude,

$$T(\phi, z)=T_1\,[\cos(2\phi)\exp(-z/H)] + T_2$$

where H is a constant tropopause height level, and T_1 and T_2 are constants.

Explain why this is a reasonable model. Use the above expression for T in combination with the equation of state provided above, to derive an expression for the meridional density gradient $\partial\rho/\partial\phi$

d. Incorporate the result from ‘c’ into the equation derived in part ‘b’. Now integrate the equation with respect to z, to yield an ordinary 2nd order differential equation in ϕ for the meridional velocity field v

e. There should be a single constant of integration in your result in part ‘d’. Determine the constant of integration such that the 2nd derivative of v with respect to ϕ vanishes at some altitude $z=z_o$

f. Focusing on one (Northern) of the two hemispheres, and assuming "no normal flow" boundary conditions such that v = 0 at latitudes ϕ =0 and ϕ=90N, explain why the solution to the remaining 2nd order differential equation implies a thermally-driven circulation [*hint*: *you needn't actually solve the differential equation! You can reason what the solution must look like for* $z<z_o$ *and* $z>z_o$ *given your expression for the 2nd derivative of v with respect to* ϕ*, the above boundary conditions, and continuity considerations, similar to what we did in class*].