combined. This is the *Stommel model* of the wind-driven circulation (Stommel, 1948). Analogous analytic models incorporating the effects of lateral friction and weak nonlinearity have been examined by Munk (1950) and Charney (1955a). The earliest numerical extensions of this work were due to Bryan (1963).

11.4.2 Box models of the thermohaline circulation

The simple models for the wind-driven circulation described in the previous section assume that buoyancy variations play little or only a passive role in the dynamics. Despite this drastic assumption, they are quite successful in representing the basic patterns of the circulation and provide the underpinning of much of the theory of the ocean general circulation. For climate applications, however, it is the spatial and temporal variations of the temperature and salinity distributions of the ocean and its capacity for heat storage and transport that are of primary concern. The determination of the thermohaline (joint effects of heat and salt on buoyancy) driven circulation and the reciprocal effects of the circulation on the distribution of water mass properties are difficult problems for several reasons. First and foremost is the essential nonlinearity of the system. The models of the previous section could be obtained through a systematic scale analysis and linearization of the governing equations. In considering the thermohaline circulation, the advection of heat and salt by the circulation is central to the problem and cannot be neglected. A further complication arises from the difference between the form of the surface forcing for temperature and salinity, as discussed in Sec. 11.2.3. As a result of the different mathematical structure of the boundary conditions on heat and salt (often referred to in the literature as mixed boundary conditions) the problem cannot be reduced to one in a single buoyancy variable. Additional complications in modeling the thermohaline circulation arise from the nonlinear equation of state for sea water and the presence of double-diffusive phenomena.

An alternative to the formal mathematical derivation of simplified models from the full equations of motion is to pose a conceptual model or simple physical analog for the system or processes being considered. This approach has been fruitfully exploited in developing our understanding of the dynamics of the thermohaline circulation. The consequences of the difference in the nature of the feedbacks between surface temperature and salinity and their respective surface forcings were first explored by Stommel (1961) using a very simple model consisting of two well-mixed reservoirs connected by pipes. In particular, he showed that the thermohaline circulation may have multiple equilibria for a given surface forcing distribution.

These ideas are illustrated using a slightly modified version of the model (Fig. 11.2), as described by Marotzke (1989). The two reservoirs are taken to represent equatorial and polar regions of the ocean. In light of the discussion

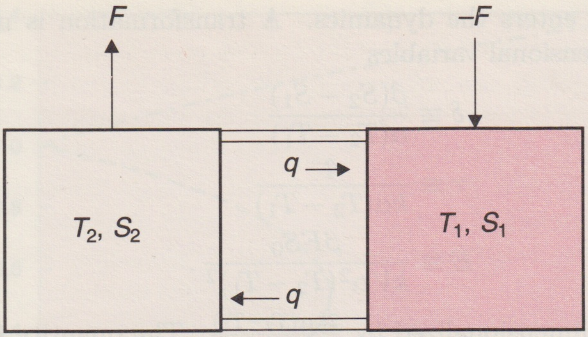

Fig. 11.2 Schematic of the two-box model of the thermohaline circulation. Box 1 represents high-latitude conditions, box 2 represents low-latitude conditions.

of surface boundary conditions above, it is assumed that the temperatures remain very close to the imposed atmospheric values T_1 and T_2 ($T_1 < T_2$) and are simply held fixed. Salinity, on the other hand, is forced by a flux of moisture F through the atmosphere from the low-latitude to the high-latitude box. The sense of the salinity forcing is to make the low-latitude box saltier and more dense and the high-latitude box fresher and less dense. The resulting torque thus opposes that of the imposed temperature differences. Conservation of salt in the boxes is given by:

$$V\frac{dS_1}{dt} = -FS_0 + |q|(S_2 - S_1) \tag{11.33}$$

$$V\frac{dS_2}{dt} = FS_0 + |q|(S_1 - S_2)\ , \tag{11.34}$$

where V is the volume of the boxes (assumed equal), S_0 is a (constant) reference salinity, and q is the rate of volume exchange in the pipes. The absolute value in the advective term arises because the same exchange is affected irrespective of the direction of the flow. The flow is driven by the pressure difference between the boxes (linearly proportional to the density difference) and retarded by friction in the pipes, and is assumed to be in instantaneous balance. This simplified dynamics is modeled using a resistivity k^{-1} to obtain:

$$q = -\frac{k}{\rho_o}(\rho_2 - \rho_1)\ . \tag{11.35}$$

The system is closed with a linear equation of state:

$$\rho = \rho_o(1 - \alpha T + \beta S) \tag{11.36}$$

where α is the thermal expansion coefficient and β is the haline contraction coefficient.

From (11.33) and (11.34) the total salt content of the system is conserved, and from (11.35) only the difference between the temperature and salinity of the two boxes enters the dynamics. A transformation is made to the following nondimensional variables

$$\delta \equiv \frac{\beta(S_2 - S_1)}{\alpha(T_2 - T_1)} \tag{11.37}$$

$$r \equiv \frac{q}{k\alpha(T_2 - T_1)} \tag{11.38}$$

$$E \equiv \frac{\beta F S_0}{kV\alpha^2(T_2 - T_1)^2} \, . \tag{11.39}$$

Time is also nondimensionalized by $\frac{2k\alpha(T_2-T_1)}{V}$. The quantity δ represents the relative contributions of salinity and temperature to the buoyancy difference between the boxes, r represents the strength of the flow relative to the purely thermally driven system, and E represents the strength of the salinity forcing relative to advection. Subtracting (11.33) from (11.34) and substituting (11.37–11.39) gives

$$\dot{\delta} = E - |r|\delta \, , \tag{11.40}$$

and (11.35) becomes

$$r = 1 - \delta \, . \tag{11.41}$$

The three steady state solutions of (11.40–11.41) are given by

$$\delta_1 = \frac{1}{2}(1 - \sqrt{1 - 4E}) \tag{11.42a}$$

$$\delta_2 = \frac{1}{2}(1 + \sqrt{1 - 4E}) \tag{11.42b}$$

$$\delta_3 = \frac{1}{2}(1 + \sqrt{1 + 4E}) \tag{11.42c}$$

and are shown in Fig. 11.3. The system can support multiple equilibria for $E < \frac{1}{4}$; there are three solutions for a given value of E. The solution δ_2 can be shown to be unstable and hence not realizable in the real climate or a numerical model.

Solution δ_1 represented by the solid portion of the curve in Fig. 11.3, has $0 \leq \delta \leq 0.5$; i.e., the contribution of temperature to the buoyancy difference dominates that of salinity. The flow is relatively strong ($0.5 \leq r \leq 1$) with "sinking" in the cold box, deep flow to low latitudes, "upwelling" in the warm box, and surface flow back to high latitudes. This corresponds to the configuration of the thermohaline circulation of the North Atlantic under present climate conditions.

Solution δ_3 , represented by the dash-dot portion of the curve in Fig. 11.3, has $\delta \geq 1$; i.e., the contribution of salinity to the buoyancy difference dominates over that of temperature. In this regime, the flow is reversed from the previous case ($r < 0$) with "sinking" in the warm box, deep flow from low to high latitudes, "upwelling" in the cold box, and surface flow from high back to low latitudes. There is evidence that such a circulation with warm

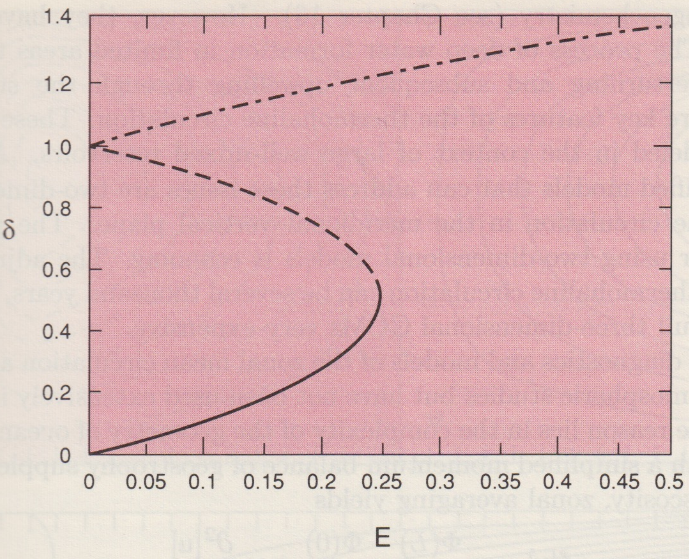

Fig. 11.3 Solution to the two-box model. The solid portion of the curve corresponds to Eq. (11.42a), the dashed portion to (11.42b), and the dash-dot portion to (11.42c).

salty deep water may have existed in the geologic past. In the forcing regime, where multiple equilibria are supported, the salinity dominated "inverse" circulation is weaker than the thermally dominated solution. This can be understood physically in terms of the relationship between residence time of water parcels in the boxes and the effect of the flux boundary condition on salinity. The longer a parcel remains in one box or the other, the more its salinity will be changed by the constant input or removal of fresh water through the surface. The slower circulation thus allows the salinity difference to build up and eventually dominate over temperature.

Welander (1986) has extended this system by adding a third box representing a second polar region. He shows that the number of stable equilibria increases to four, corresponding to combinations of the solutions of two independent two-box models: a symmetric solution with sinking in the polar boxes and upwelling in the equatorial box, a symmetric solution with sinking in the equatorial box and upwelling in the polar boxes, and two asymmetric solutions with sinking in one of the polar boxes and upwelling in the other polar box. The existence of multiple equilibria has also been found in more complex ocean models and coupled ocean–atmosphere models as described further in the following sections and in Chapter 17.

11.4.3 Two-dimensional meridional plane models

Box models provide a means of exploring basic physical processes active in the thermohaline circulation, and have been particularly useful