Localized Storm Tracks in the Absence of Local Instability

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ABSTRACT

An idealized, linear barotropic model on an f plane is used to demonstrate the existence of a zonally localized storm track in the absence of any kind of local instability; with a background flow consisting of two jets separated by a local minimum, two distinctly localized eddy streamfunction variance (and eddy kinetic energy) peaks emerge on either side of the local jet minimum. In contrast, the decrease of the eddy vorticity variance at the jet minimum is essentially negligible. As the stretching deformation field of the background flow strengthens, the transient eddy is deformed irreversibly in the deformation region. The resulting enstrophy cascade toward smaller scale plays an important role in terminating the model’s storm tracks, and in this case the second storm track downstream of the local jet minimum is substantially weakened.

1. Introduction

Following the observational study of Blackmon (1976) and Blackmon et al. (1977), regions of enhanced baroclinic wave activity, particularly in the Northern Hemisphere (NH) region, known as the two oceanic (Pacific and Atlantic) storm tracks have been the subject of numerous studies. Because both storm tracks lie slightly downstream of the locally enhanced baroclinicity zone associated with the climatological jets, it was tempting to understand the storm track dynamics in terms of absolute instability associated with the local baroclinicity maxima (Pierrehumbert 1984). Lin and Pierrehumbert (1993) also suggested a mechanism for significant local amplification of a convectively unstable mode if the mode propagates slowly enough so that it stays in the unstable region for a sufficiently long time. In either case, the presence of the locally enhanced baroclinicity in the background flow is the essential requirement.

Analogous to the much better understood baroclinic wave life cycle in a zonally homogeneous flow (Simmons and Hoskins 1978; Randel and Stanford 1985; Feldstein and Held 1989), the above theories fit into a picture of baroclinic growth followed by barotropic decay in the upstream and downstream ends of the storm track, respectively. Although the results of eddy kinetic energy budgets (e.g., Lau 1979; Kushner and Esbensen 1986) seemed to support this picture, with positive baroclinic energy conversion upstream and negative barotropic energy conversion downstream of the storm tracks, diagnostics by the eddy kinetic energy budget involve ambiguities due to the conversion terms between the eddy kinetic and potential energy, and between the eddy potential and the mean potential energy, etc. (Plumb 1983). This problem can be resolved if a conservation law of wave activity is used instead. Andrews (1983) and Plumb (1985) formulated conservation laws for small-amplitude transients on a wavy mean flow, but they have not proven to be of much practical use.

Using an approximate conservation law, Plumb (1986) diagnosed the Northern Hemisphere winter time transient eddy wave activity. There is a large zonal component of the total eddy activity flux \( M_R \) (in his notation) for the synoptic-scale eddies (Fig. 7 in Plumb) in the upper troposphere, upstream of both the Atlantic and Pacific storm tracks as well as upward fluxes in both storm tracks. As with the Eliassen–Palm (EP) flux (Eliassen and Palm 1961; Edmon et al. 1980), the upward flux represents a poleward heat flux. The zonal component of the eddy activity flux in the upper troposphere, upstream of the two oceanic storm tracks, is an unexpected feature from the viewpoint of the simple baroclinic life cycle involving local, absolute instability where the wave activity grows and decays in the upstream and downstream ends of the storm tracks, respectively. However, this observation should be taken with caution as Plumb (1986) pointed.
out that the quality of data used may not be good enough for the eddy enstrophy statistics.

In light of the existence of coherent baroclinic wave packets in both observations and models (Lee and Held 1993), one might expect that this zonal component of wave activity flux in the upper troposphere may be related to the coherent, eastward-propagating wave packets. Indeed, Higgins and Schubert (1993) found a “seesaw” relation between the Pacific and Atlantic storm tracks in a GCM and also showed that they are related to coherent baroclinic wave packets. The wave packets are also present in the Northern Hemisphere winter, as shown in European Centre for Medium-Range Weather Forecasts (ECMWF) analyzed data by Chang (1993). In Fig. 2 of Chang, one sees that the wave packets amplify in the vicinity of the two oceanic storm tracks, but they usually do not disappear between the storm tracks, especially over the eastern Pacific and North America. While the life cycle of baroclinic waves is certainly a plausible mechanism, the existence of coherent wave packets strongly suggests that it can be only part of the story for the existence of storm tracks.

As the wave packets traverse stationary waves in the NH winter, one can conjecture both baroclinic and barotropic modulations of the wave packets. The baroclinic modulation can be thought of as Type B cyclogenesis (Petterssen and Smeybe 1971; Whitaker and Barcilon 1992). As stated earlier, the upward wave activity flux in Plumb (1986) represents a poleward heat flux, but this flux does not distinguish Type A cyclogenesis from Type B cyclogenesis. Therefore, it is possible that part of the upward wave activity flux in the storm track region represents the baroclinic modulation.

The structure and maintenance of upper-tropospheric disturbances is one of the central features in the atmospheric general circulation. Because barotropic processes cannot be ignored in the upper troposphere, we feel that it is also important to understand the barotropic modulation of the synoptic-scale disturbances. In fact, there is ample evidence that in the middle and upper troposphere the synoptic-scale waves are strongly modulated by monthly mean, or quasi-stationary, waves (Mullen 1987; Dole 1986; Lau 1988). These studies clearly show that the eastward propagation of the synoptic-scale eddies are blocked by quasi-stationary ridges, lying downstream of the two oceanic storm tracks.

In this paper, we examine this barotropic modulation of the wave packets, in the absence of barotropic and baroclinic instability. In order to eliminate baroclinic modulation, we use a barotropic model. The synoptic disturbances in this barotropic model are mimicked by a wave maker with an appropriate frequency and zonal wavenumber. It is obvious that the wave maker is a very crude representation of the baroclinic waves, yet it allows one to examine the barotropic modulation in a very simple context. In the present study, we do not address the generation and maintenance of either the wave packet (Lee and Held 1993) or the background stationary flow (Held 1983; Nigam et al. 1988), but we simply model the observed wave packets and the stationary wave, and examine the linear problem, that is, the modulation of the synoptic disturbances for a fixed background flow.

2. Model description

A nondimensionalized, barotropic vorticity equation on an $f$ plane is

$$\frac{\partial q}{\partial t} = -J(\phi, q) - \nu \nabla^2 q + F,$$

(2.1)

where $q = \partial \psi / \partial x - \partial \theta / \partial y = \nabla^2 \phi$, $J(\cdot)$ is the Jacobian, and $\nu$ the biharmonic diffusion coefficient. The vorticity forcing is represented by $F$ on the rhs of (2.1). The velocity and length scales are nondimensionalized by typically observed values of zonal wind velocity $U_0$ and length scale $\lambda$ in the upper troposphere. Therefore, the nondimensional quantities $(u, L, t) = (u^* / U_0, L^* / \lambda, t^* / \lambda)$, with the asterisk representing the dimensional quantities.

The $f$-plane approximation is used instead of the $\beta$-plane approximation so that the wave maker will generate disturbances that resemble coherent wave packets. It should be stated that the $\beta$ effect is dynamically important because individual waves in the wave packets decay barotropically as they disperse meridionally (Lee and Held 1993). At the same time, an eastward ageostrophic energy flux, along with baroclinic wave growth, maintains the wave packets’ coherence. Clearly, these delicate wave packet dynamics are not realized in the wave maker, but the $f$-plane approximation allows the artificial waves to propagate eastward without losing strength as the coherent wave packets in the atmosphere do. While the $f$-plane approximation is a rather severe idealization, because our aim here is to examine the kinematic modulation of the transient waves, we do not believe that the lack of the dispersion due to the earth’s rotation effect is crucial.

We decompose the flow into a time-invariant basic state and a perturbation from the basic state: $q = Z + \zeta$, $\phi = \Psi + \psi$ and $F = H + h$, where the uppercase letters represent the basic flow and the lowercase letters the deviation from the basic flow. By linearizing Eq. (2.1) about a time-invariant basic state, and by subtracting $-J(\Psi, Z) - \nu \nabla^2 Z + H$ from the rhs of (2.1) and neglecting $J(\psi, \zeta)$, we obtain a linear perturbation equation,

$$\frac{\partial \zeta}{\partial t} = J(\Psi, \zeta) + J(\psi, Z) - \nu \nabla^2 \zeta + h.$$

(2.2)
We divide \( h \) into two terms; that is, \( h = h_1 + h_2 \), with \( h_1 \) and \( h_2 \) representing the wave maker and a sponge layer, respectively. Because the lateral boundary condition is periodic in \( x \), in the absence of a sponge layer, recycling waves can interfere with the wave maker itself.

This model is finite differenced meridionally and truncated spectrally in the zonal direction. The width and length of the channel are \( 4\pi \) and \( 20\pi \), respectively. There are 120 grid points in \( y \), and 80 zonal waves with zonal wavenumbers \( 0.1, 0.2, 0.3, \ldots, 8.0 \). The time stepping is leapfrog, and the model is periodically re-started with the Euler backward scheme to remove the effect of the computational mode. The value of \( \nu \) is \( 10^{-8} \).

The structure of the basic flow and the wave maker are very similar to that in the blocking study of Shufts (1983), except we prescribe vorticity instead of the streamfunction in order to make the vorticity field continuous everywhere in the domain. This change makes interpretation of the model results easier. Although we do not show it in this paper, we obtain essentially the same results using a basic flow prescribed by the streamfunction.

The vorticity of the basic flow \( Z(x, y) \) is

\[
Z(x, y) = -\frac{2\pi A}{W} \sin \left( \frac{2\pi y}{W} \right) , \\
x < x_-, \quad x > x_+ \tag{2.3a}
\]

\[
Z(x, y) = -\frac{2\pi A}{W} \sin \left( \frac{2\pi y}{W} \right) \cos \left( \frac{2\pi(x - x_-)}{W} \right) \\
\times \exp \left[ \frac{(x - x_-) \ln 2}{W/2} \right] , \quad x_- < x < x_0 \tag{2.3b}
\]

\[
Z(x, y) = -\frac{2\pi A}{W} \sin \left( \frac{2\pi y}{W} \right) \cos \left( \frac{2\pi(x - x_-)}{W} \right) \\
\times \exp \left[ \frac{(x_+ - x) \ln 2}{W/2} \right] , \quad x_0 < x < x_+ \tag{2.3c}
\]

where \( W \) is width of the channel, \( x_0 = (x_- + x_+)/2 \) with \( x_+ - x_- = W \), and \( A \) is a constant. In Fig. 1, the structure of \( Z(x, y) \), is shown with thin contours. Given this vorticity distribution, the vorticity gradient is not single signed; there is one slowly growing unstable mode. As we will discuss later, however, the growth rate of the unstable mode is very small and the structure of the mode does not have any resemblance to that of the wave maker. Therefore, we do not believe that the barotropic unstable mode is relevant for our calculations. In our experiment, the width of the stretching deformation region \( x_- < x < x_+ \) is chosen to be identical to the channel width \( W \), so that the deformation region occupies a square box. The value of \( W \) is fixed at \( 4\pi \). If we take the value of \( \lambda \) to be 1000 km, the value of \( W \) is \( \approx 12 \, 000 \text{ km} \) in dimensional units, and this value is close to the typically observed width of the stretching deformation region downstream (upstream) of the Pacific (Atlantic) jet. In order to set the maximum velocity in the domain as 1 and to keep the winds westerly everywhere, we add a zonally constant zonal wind, \( 1 - A \), over the entire domain:

\[
U(y) = 1 - A \int_{y}^{y'} [Z(x, y')]dy' \\
+ \frac{1}{W} \int_{y}^{y'} \int_{0}^{W} [Z(x, y')]dy'dy'', \tag{2.4}
\]

where the bold-square bracket represents the zonal mean.

The wave maker \( h_1 \) is prescribed by

\[
h_1(x, y) = \eta \sin \left[ \frac{\pi(y - y_1)}{W/4} \right] \cos \left[ \frac{3\pi(x - ct)}{L_w} \right] \sin \left[ \frac{\pi x}{L_w} \right] , \\
0 < x < L_w , \quad y_1 < y < y_2 \tag{2.5}
\]

where \( y_1 = 3W/8, y_2 = 5W/8, L_w = L_e/8, c = L_w\omega/3\pi \) with \( L_e \) being the length of the channel, and \( \eta \) is an arbitrary small constant. The domain-averaged value of \( h_1 \) is zero. This choice of parameters produces a slightly meridionally elongated eddy. Also, the zonal wavenumber is 12, so that for a realistic length of the mid-latitude belt, it is equivalent to zonal wavenumber 6, because our channel is twice the circumference of the earth at midlatitudes. The meridionally elongated eddy is chosen to mimic the structure of the high-frequency eddies, of period 2.5–6 days (Hoskins et al. 1983; Blackmon et al. 1984). The frequency is fixed as 1, giving a dimensional value of 3–4 days, if \( U_0 \approx 30 \text{ m s}^{-1} \). The structure of the wave maker is shown by thick lines in Fig. 1.

The sponge layer \( h_2 \) is given as

\[
h_2(x, y) = -D_{si} \sin^2 \left\{ \frac{0.5\pi[x - (1 - \alpha)L_e]}{\alpha L_e} \right\} \zeta , \\
(1 - \alpha)L_e < x < L_e ,
\]
\[ h_2(x, y) = 0, \text{ elsewhere,} \]  

(2.6)

where the value of \( D \) is 0.5, which is small enough to eliminate reflection. Once \( D \) is determined, we vary \( \alpha \) to choose the minimum value for which the sponge layer is long enough to prevent the waves from tunneling through. In our case, \( \alpha \) is chosen as 0.3, giving the sponge layer a length of \( 6\pi \). The stippled region in Fig. 1 represents the sponge layer.

3. Scale analysis and WKB solution

a. Scale analysis

The main point of this paper can be made by a simple scaling argument; away from the wave maker and sponge layer, and neglecting the biharmonic diffusion, if we assume \( k^2 \gg l^2 \), (2.2) becomes

\[ \frac{\partial \zeta}{\partial t} = -U \frac{\partial \zeta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial Z}{\partial y}, \]  

(3.1)

where \( \zeta = -k^2 \psi \). A further approximation can be made if the ratio of the advection of eddy vorticity by the mean flow to that of the mean flow vorticity advection by the eddy flow is large, that is, if \( k^2 \gg \left| (\partial Z/\partial y)/U \right| \). Under this condition, (3.1) simply becomes the vorticity advection equation; namely,

\[ \frac{\partial \zeta}{\partial t} = -U \frac{\partial \zeta}{\partial x}, \]  

and \( \omega = kU \). Multiplying this equation by vorticity and averaging over time, one obtains the result that the eddy enstrophy is independent of \( x \); equivalently, the magnitude of the vorticity \( |\zeta| \) should be independent in \( x \). It follows that the eddy streamfunction amplitude \( |\psi| \) and the meridional wind \( |v| \) do not have to be constant in \( x \) if the mean flow changes, because

\[ |\psi(x)| \approx k(x)^{-2} |\zeta| \approx \frac{U(x)^2}{\omega^2} |\zeta| \]  

(3.2a)

and

\[ |v(x)| \approx k(x)^{-1} |\zeta| \approx \frac{U(x)}{\omega} |\zeta|. \]  

(3.2b)

The above two relations tell us that the streamfunction and the velocity amplitude decrease as the eddy becomes more anisotropic, even when the vorticity amplitude is conserved. This is possible because neither velocity nor streamfunction is a conserved quantity. Among others, one familiar example of this sort is found in vertically propagating waves in a compressible fluid, where the density decrease with height manifests itself as streamfunction amplification with height. However, it is noteworthy that the storm track is often defined as a locally enhanced region of geopotential (or streamfunction) variance or eddy kinetic energy (EKE) (e.g., Blackmon et al. 1977; Lau 1979), and it happens that the storm track termination usually appears in the local jet minimum, in the vicinity of a strong stretching deformation field. The implication is that it is possible for the storm tracks in the NH winter to appear localized, because they are measured by non-conservative quantities.

In the following subsection, we elaborate this argument applying a formal WKB solution to (3.1). As we will see, when \( k^2 \gg \left| (\partial Z/\partial y)/U \right| \), the WKB solution yields the same interpretation based on the above vorticity advection equation. However, the formal WKB analysis provides additional insight into the wave propagation characteristics under the circumstances where the condition \( k^2 \gg \left| (\partial Z/\partial y)/U \right| \) should be relaxed.

b. WKB analysis

We first look for an asymptotic solution of the wake amplitude and zonal wavenumber for a given basic flow. Suppose that we look for a solution of the form

\[ \psi = \Re[\tilde{\psi}(x) \exp(iy - \omega t)], \]

(3.3)

where \( \tilde{\psi}(x) \) is a complex streamfunction amplitude and a function of \( x \) only, \( l(x, y) \) the local meridional wavenumber, and \( \omega \) the real frequency specified for the wave maker. For an eddy that is meridionally elongated, \( k \gg l^2 \), where \( k \) is the zonal wavenumber. In order to keep the derivation as simple as possible, we take a limit where \( l(x, W/2) = 0 \). That is, we consider an eddy that is symmetric about the center of the channel, and the ray is propagating only to the east. We note that if we look for a solution of the form \( \psi = \Re[\tilde{\psi}(x) \sin(ly) \exp(-i\omega t)] \), where \( l = \pi/W \), we once again obtain (3.7), shown below, at the center of the channel as long as \( k^2 \gg l^2 \). Under the above assumptions, for an inviscid limit, away from the wave maker and sponge layer, (2.2) becomes

\[ -i\omega \left( \frac{\partial^2 \tilde{\psi}}{\partial x^2} - \frac{\partial \Psi}{\partial y} \frac{\partial^3 \tilde{\psi}}{\partial y^3} + \frac{\partial \Psi}{\partial y} \frac{\partial^2 \tilde{\psi}}{\partial y \partial x} + \frac{\partial \Psi}{\partial y^3} \frac{\partial \tilde{\psi}}{\partial x} \right) = 0 \]  

(3.4)

at the center of the channel. The derivative in \( y \) is evaluated at \( y = W/2 \). We introduce a slow zonal scale, \( X = \epsilon x \), where \( \epsilon \ll 1 \), and the basic state is only a function of the slow zonal scale \( X \), and \( y ; \Psi = \Psi(X, y) \). This means that the zonal scale of the stretching deformation region is much greater than the zonal scale of the transient waves. In this limit, (3.4) can be written as

\[ -i\omega \left( \frac{\partial^2 \tilde{\psi}}{\partial x^2} - \frac{\partial \Psi}{\partial y} \frac{\partial^3 \tilde{\psi}}{\partial y^3} + \frac{\partial \Psi}{\partial y} \frac{\partial^2 \tilde{\psi}}{\partial y \partial x} + \frac{\partial \Psi}{\partial y^3} \frac{\partial \tilde{\psi}}{\partial x} \right) = 0. \]

(3.5)

Now we look for a standard WKB solution (e.g., see Bender and Orszag 1978) of the form
\[ \dot{\psi} \approx \exp \left[ \frac{S_0(X)}{\epsilon} + S_1(X) + \epsilon S_2(X) + \cdots \right], \quad (3.6) \]

where \( S_i(X), \, i = 0, 1, 2, \ldots, \) is complex. Substituting (3.6) into (3.5), and assuming that \( S_i(X) \) and its derivatives are order unity, equations for the lowest two orders are

\[
\begin{align*}
\text{O}(1): & \quad U_{xx} S_0' + i \omega S_0'' - U S_0'' = 0 \quad (3.7a) \\
\text{O}(\epsilon): & \quad -i \omega (2 S_0 S_1' + S_0'') \\
& \quad + 3U(S_0'' S_0'' + S_0'' S_1') - U_{xx} S_1' = 0, \quad (3.7b)
\end{align*}
\]

where \( S' = dS/dX, \quad S'' = d^2S/dX^2, \) etc. We first consider solutions for (3.7a). If \( \omega^2 > 4UU_{xy} \), that is, for high-frequency eddies and for moderate meridional variation of the basic zonal wind, two nontrivial solutions for (3.7a) are

\[
k_{\pm} = -i S_0' = \frac{1}{2} \left[ \frac{\omega}{U} \pm \left( \frac{\omega^2}{U^2} - 4 \frac{U_{xy}}{U} \right)^{1/2} \right]. \quad (3.8a,b)
\]

The solution \( k_{\pm} \) allows westward as well as eastward propagation, whereas \( k_{+} \) represents an eastward-propagating wave. Because we are interested only in eastward-propagating solutions, we consider the case of \( k = k_{+} \).

Given \( S_0'(X) \), and therefore \( k(X) \), the solution for \( S_1'(X) \) is obtained by solving (3.7b):

\[
S_1' = \frac{(\omega - 3Uk) \frac{dk}{dX}}{U_{xy} + (3uk - 2\omega)k}. \quad (3.9)
\]

We now know from (3.8a) and (3.9) that \( S_0'(X) \) is imaginary and \( S_1'(X) \) is real. Given this information, we rewrite (3.6) for clarity:

\[
\dot{\psi}(X) \approx \exp \left[ \frac{ik(X)X}{\epsilon} \right] \exp \left[ \int_0^X \frac{dS_1(X*)}{dX*} \, dX* \right], \quad (3.10)
\]

where the first and the second exponents represent phase and amplitude, respectively. For \( \omega^2 \gg |4UU_{xy}| \) (3.8a) reduces to \( k(X) = \omega/U \). We note that the condition \( \omega^2 \gg |4UU_{xy}| \) is equivalent to \( k^2 \gg |(\partial Z/\partial y)/U| \) in the scaling argument discussed earlier pertaining to (3.1), except for the factor of 4. Substituting \( k(X) = \omega/U \) and neglecting \( U_{xy} \) in (3.9), (3.10) becomes

\[
\dot{\psi}(X) \approx \exp \left( \frac{ik(X)X}{\epsilon} \right) |\psi(X)|, \quad (3.11)
\]

where

\[
|\psi(X)| \approx \left[ \frac{k(0)}{k(X)} \right]^2 |\psi(0)|. \quad (3.12)
\]

So, when \( \omega^2 \gg |4UU_{xy}| \), there is a simple relation between the streamfunction amplitude and its zonal scale; the streamfunction amplitude decreases as a power of 2 as the wavenumber increases.

4. Numerical experiments

In this section, we describe results of a numerical integration of the linear barotropic vorticity equation (2.2) and compare those with the analytical results obtained in section 3. For each experiment, the time-mean statistics are obtained by averaging over the last 60 days of a 100-day integration. The first experiment is for a zonally uniform basic state: \( U = 1, \, V = 0 \). In Fig. 2a, a snapshot of the streamfunction is shown after the flow has reached a steady state. In the absence of surface friction, the waves are simply advected by the basic zonal flow, keeping their strength until they are absorbed by the sponge layer at the right end of the channel. The result is a single storm track filling the entire domain, as shown in Fig. 2b.

a. “Realistic” deformation case; \( A = 0.3 \)

The second experiment is for a basic flow with a stretching deformation field. The total streamfunction field of the basic flow \( \Psi \) is shown in Fig. 3a. The corresponding basic zonal wind \( U \) has a local minimum along the center of the channel, at \( x = 0.38L_x \) (contours in Fig. 12a). The value of the minimum \( U \) is 0.4, as determined by the constant \( A \) in (2.3). Note that the maximum wind speed in the domain is always fixed at 1. Thus, the ratio of the minimum to the maximum zonal wind is 0.4, and this value is close to that observed in the NH upper-tropospheric winter climatology, particularly between the Pacific and Atlantic jets, in the Western Hemisphere (Blackmon et al. 1977; Hoskins et al. 1989). We call this case “realistic,” for convenience. In Figs. 3b and 3c, we show the stretching and shearing deformation fields, respectively. Notice
that the stretching deformation field is confined to the vicinity of the jet exit and jet entrance, with its maximum value of 0.234 ($\approx 10^{-6}$ s$^{-1}$ in dimensional units). In these regions, the shearing deformation is relatively weak. We also note that barotropic instability is irrelevant for this calculation, because the linear growth rate is very small, being essentially neutral as far as our calculations are concerned.

Figure 4 shows snapshots of streamfunction at various stages of integration. As the wave enters the jet exit, it is meridionally stretched and its amplitude also decreases (Fig. 4a; $t = 25$). As the wave reenters the jet downstream (Fig. 4b; $t = 33$) it shrinks meridionally and its amplitude increases, recovering its upstream characteristics, although slight structural differences in the eddies can be seen between the two jet regions, as can be seen in Fig. 4c, a typical snapshot of the streamfunction after the flow reached a statistically steady state. Stated another way, waves undergo a reversible stretching process as they propagate through the deformation region. The standard deviation of the streamfunction is shown in Fig. 5a, revealing two distinct localized storm tracks separated by the jet minimum. It is important to emphasize that these two storm tracks occur in the absence of any local instability. Because of the wave maker, one may argue that the first storm track can be thought of as a result of local instability, but the second storm track appearing downstream of the jet minimum is clearly free from such local instability. Comparing with Fig. 2b, it is clear that the local jet minimum and the stretching deformation field are responsible for the local minimum streamfunction variance in this case.

The local zonal wavenumber $k(x)$ and the streamfunction amplitude $|\psi|$ normalized by the maximum amplitude at the center of the channel are shown in Fig. 6 together with the WKB solution (3.10). Because $kU \approx \omega$ in the WKB analysis, one can also relate $|\psi|$ directly to the basic zonal wind $U$, as shown in (3.2a). The numerical solution for $k(x)$ is computed graphically; for each model day and for each latitude, we compute the distance between one peak and the next peak of $|\psi|$, and from this distance, $\Delta L(x)$, the local zonal wavenumber is readily obtained by $k(x) = \pi/\Delta L(x)$. There are discrepancies between the numerical and the WKB solution, yet considering the approximations used to obtain the WKB solution, the agreement is satisfactory. In the same figure, we also plot the solution for the streamfunction amplitude from (3.12). This solution also agrees well with the full WKB solution where the value of $|4UU_{xy}|$ is about 0.3, indicating that for a high-frequency eddy with a moderate value of $|4UU_{xy}|$, a rather simple, intuitive argument is able to explain the transient streamfunction amplitude change.

The EKE also exhibits two local maxima (Fig. 5b), although they appear less localized than the stream-
function amplitude. In contrast with the behavior of streamfunction or EKE, the variation of vorticity amplitude with the varying basic flow is negligible. In Fig. 4d, a snapshot of vorticity is shown. As for the streamfunction (Fig. 4c), the vorticity is stretched meridionally in the region of jet minimum, but its amplitude does not decrease in this region (see also Fig. 5c). This is consistent with what (3.12) tells us, and therefore the agreement between the numerical results and (3.12) as demonstrated in Fig. 6 comes as no surprise.

b. Stronger deformation case; $A = 0.4$

In this subsection, we describe the behavior of transient waves with a somewhat stronger basic deformation field. Figure 7a shows the total streamfunction in this case. In this calculation, we set $A = 0.4$ in (2.3), resulting in a 25% stronger stretching deformation rate than that of the realistic case in section 4a. We only explore the sensitivity of transient waves for a stronger deformation rate, because for a weaker deformation rate the results are somewhere between that for the uniform basic flow and that for the realistic deformation case.

Figures 7b and 7c show snapshots of the wave streamfunction and vorticity after the flow reached a statistically steady state. Both amplitude and structure of these quantities change irreversibly in the deformation region. As a result, the standard deviation of streamfunction (Fig. 8a) shows only a single storm track, instead of two as in Fig. 5a. The change of the standard deviation of vorticity (Fig. 8b) is also striking, with its amplitude decreasing sharply at that region. Clearly, this behavior cannot be explained by the WKB solution. One obvious candidate that can explain the irreversible process in this linear model is the subgrid-scale dissipation. An ideal way to investigate the generation and termination of the storm track is to use an appropriate conservation law. For the zonally varying basic state, diagnostics analogous to that proposed by Plumb (1986) seem to be suitable for this purpose. But we find it awkward to use in our case because there are four points in the domain where the basic-state potential vorticity gradient vanishes because $\beta = 0$, yet the enstrophy has a finite value, resulting in infinite wave activity density at those points.

Although there is no reason to believe that the transient enstrophy is a conserved quantity, because it is essentially conserved for the weaker deformation case (Fig. 5c) on the $f$ plane, it will be useful to examine the dissipation in the context of an enstrophy budget. The transient eddy enstrophy equation is

$$\frac{\partial e}{\partial t} = -\mathbf{\nabla} \cdot \mathbf{\nabla} e - \mathbf{\nabla} \cdot \mathbf{\nabla} Z - \frac{\zeta}{\nu} \nabla^4 \zeta,$$

where $e = \overline{\zeta^2}/2$. The overbar represents the time average. The last two terms on the rhs are shown in Figs. 9a and 9b, respectively. In these figures, we show only

![Graph showing local zonal wavenumber and eddy streamfunction amplitude](image-url)

FIG. 6. The local zonal wavenumber and eddy streamfunction amplitude from the WKB and numerical solution at the center of the channel. Also shown is an approximate solution of eddy streamfunction amplitude from (3.12).
the part of the domain away from both the wave maker and the sponge layer to focus on the dissipation due to horizontal diffusion in the "interior" (we use this term hereafter). Figure 9a indicates that the vorticity flux is mostly upgradient, except in a small region near the jet entrance. Although it is hard to see in Fig. 8b, the eddy enstrophy does decrease with increasing $x$ in the region $x < 5\pi$. Therefore, the upgradient vorticity flux is well balanced by the advection of the eddy enstrophy by the basic flow in this region. At least in the vicinity of the jet exit and entrance region, the vorticity flux seems to show a gross similarity with the observational study of Illari and Marshall (1983) and the model results by Shutts (1983), in which the basic state satisfies a free vorticity equation. As discussed in section 2, in order for the winds to be westerly everywhere, we added the constant $1 - A$ to the basic-state zonal winds, as shown in (2.4). Because our model is on an $f$ plane, rather than on a beta plane as in Shutts (1983), the presence of this constant prevents the basic streamfunction from satisfying the free vorticity equation. Therefore, we do expect that the details of the three terms on the rhs of (4.1) in our calculation will be different from those in Shutts (1983). However, the main feature, that is, the termination of the storm track at the jet minimum, or block region, is seen in both calculations.

An interesting feature in Fig. 9b is that the maximum enstrophy dissipation (shaded area) occurs in the jet entrance region. This is rather counterintuitive, because the barotropic decay occurs in the jet exit, where subsequent small-scale generation is expected due to wave breaking. We also note that in the jet entrance region the dissipative effect dominates over the effect of the vorticity fluxes (compare Figs. 9a and 9b). Whether the strong dissipation occurs in the jet exit region, as in Shutts (1983), or in the jet entrance region, as in our case, the central point is that the deformation can terminate a storm track through the enstrophy cascade.

**Fig. 7.** Basic-state (a) total streamfunction $\Psi$, snapshots of the (b) eddy streamfunction and (c) eddy vorticity at $r = 70$ for the stronger deformation rate case (A = 0.4). The contour interval for (a) is 0.05; contour intervals for (b) and (c) are arbitrary.

**Fig. 8.** Standard deviation of the (a) eddy streamfunction and (b) eddy vorticity for $A = 0.4$. Shading denotes values greater than 85% of the maximum.

**Fig. 9.** (a) The eddy enstrophy source or sink $-\vec{v} \cdot \nabla Z$, the second term on the rhs of (4.1), and (b) the eddy enstrophy dissipation by the biharmonic diffusion $-\nu Z Z Z$, the third term on the rhs of (4.1), for $A = 0.4$. The contour intervals for (a) and (b) are the same.
For the weaker deformation case, examined earlier, the diffusive dissipation is negligible compared with the first and second terms on the rhs of (4.1) (not shown), and its maximum dissipation rate is an order of magnitude smaller than that for the stronger deformation case. The rate of dissipation increases an order of magnitude as the deformation rate increases by only 25%. One possible explanation for the dramatically increased dissipation may be the eddy-straining mechanism (Shutts 1983) and/or critical-layer absorption.

In order to investigate the possible critical-layer absorption, we first compute the phase speed of the transient streamfunction. Although the problem at hand deals with a zonally asymmetric basic flow, due to the lack of theory for critical-layer dynamics in a general basic flow, we attempt to understand the critical-layer dynamics in the context of a zonally symmetric framework. This means that we are concerned only with meridionally propagating waves incident on the critical layer parallel to the zonal direction, at least locally.

Because the waves are generated by a wave maker with a fixed frequency, once the local zonal wavenumber is known, the phase speed \(c(x) = \omega/k(x)\) is readily obtained. The method for calculating \(k(x)\) is described earlier. Figure 10a shows \(U - c(x)\), where the thick solid line indicates the zero contour. Except for the upstream side of the deformation region, it seems that the zero line parallel to the zonal direction in general corresponds well with the location of strongest dissipation (Fig. 9b). In particular, we focus on the jet entrance region enclosed by a box in Fig. 10a, where the critical line is parallel with the zonal direction and dissipation is enhanced (Fig. 9b). The structure of the meridional momentum flux (Fig. 10b) is also consistent with the critical layer, showing a momentum flux jump along the layer. A meridional cross section at the jet entrance region, indicated by an arrow in Fig. 10b, is shown in Fig. 10c. At \(y = 0.45\) and 0.55 W, the sign of the momentum flux changes abruptly, with a region of essentially zero value between the two latitudes. Similar pictures as shown in Fig. 10c for other longitudes also show a momentum flux jump near \(y = 0.25\) and 0.75 W, but not near \(y = 0.45\) and 0.55 W. These two latitudes correspond well with the critical lines inside the box in Fig. 10a and the enhanced dissipation region in Fig. 9b.

Although it is impossible to draw any concrete assertion from the above analysis because it is not based on a precise theory, the analysis sheds light on how the dissipation can be enhanced in the jet entrance region possibly due to critical-layer absorption. It is then a natural question to ask what makes a critical layer occur only in the jet entrance and not in the jet exit region, in this particular case. Figure 7c shows that the structure of the eddy vorticity is quite different between these two regions; in the jet exit the vorticity contours are not yet strongly elongated meridionally, but the vorticity contours entering the jet entrance are more severely elongated as they propagate through the jet minimum. This difference is, at least partially, due to the strong advection in this idealized model; if the advection effect is large, the residence time for the eddies in the jet exit region will be short, and therefore, there will not be enough time for significant straining to occur. Clearly, the more strongly elongated, the better the eddies “feel” the background shear, resulting in strong meridional propagation in the jet entrance region. As these eddies are tilted toward the zonal direction, the zonal scale of the eddies increases, and therefore \(c(x)\) decreases, inducing the critical layer. However, we point out that there are at least three caveats in the critical-layer interpretation for the enhanced dissipa-
Fig. 11. A schematic picture of the structure of the eddies near the shaded area in Fig. 10b.

The change of the eddy structure in the vicinity of the stretching deformation field is also seen in the observations. Lau (1988) examined empirical orthogonal functions (EOFs) of the NH storm tracks using 19 years of data, and showed high and low composites of the six leading EOFs (three from the Pacific, and three from the Atlantic storm tracks) and corresponding monthly mean 500-mb geopotential height. In cases of strong quasi-stationary ridges, the horizontal components of the $E$ vector of Trenberth (1986), $E = [(v^2 - u^2)/2, -u\bar{u}]$, indicate that the synoptic-scale eddies become zonally elongated downstream of the quasi-stationary ridge. In Fig. 12b, we show $E$ vectors along with the basic zonal wind. Similar to the results of Lau (1988), in the jet entrance the $E$ vectors point westward, indicating that the eddies became zonal in that region. For the weaker deformation rate case, examined in section 4a, the $E$ vector points eastward everywhere (Fig. 12a), although the length of the vector

$$\delta y\delta n \approx (\delta y^2 + \delta U^2 \delta t^2)^{1/2} \delta n', \quad (4.2)$$

where $\delta n$ and $\delta n'$ are scales of the eddies normal to the stretching axis of the unstrained and stretched eddies, respectively. So the smallest scale of the eddies are measured by $\delta n'$. One also can write $\delta U = (\delta U/\delta y)\delta y$, and substituting this in (4.2), we can write $\delta n'$ as

$$\delta n' \approx \delta n/[1 + (\delta U/\delta y)^2(\delta x/U)^2]^{1/2}, \quad (4.3)$$

where we used $\delta t = \delta x/U$. In the background flow that we used, the deformation scale is the same both in the $x$ and $y$ directions; that is, $\delta x = \delta y$, and therefore (4.3) reduces to $\delta n' \approx \delta n/[1 + (\delta U/U)^2]^{1/2}$. Because the enstrophy dissipation is parameterized by the biharmonic diffusion, the dissipation rate of the stretched eddy normalized by the unstretched eddy is $(\delta n/\delta n')^4 \approx [1 + (\delta U/U)^2]^2$. For the realistic case, $\delta U = 0.6$ and $U = 0.4$ so that $[1 + (\delta U/U)^2]^2 \approx 11$; for the stronger case, $\delta U = 0.8$ and $U = 0.2$ giving $[1 + (\delta U/U)^2]^2 \approx 289$. This rough estimation also gives more than an order of magnitude increase in the dissipation rate with only a 25% change in the deformation rate.

Fig. 12. Basic-state zonal wind (solid contours) and $E$ vectors $E = [(v^2 - u^2)/2, -u\bar{u}]$ for (a) $A = 0.3$ and (b) $A = 0.4$. In (b), the arrows in the jet entrance region ($7.5\pi < x < 12.5\pi$) are enlarged in a separate frame. The contour interval of the zonal wind is 0.1, and the vector length is arbitrary.
tor changes near the jet minimum. This behavior is similar to some of the weak quasi-stationary ridge cases in Lau (1988).

5. Concluding remarks

Barotropic modulation of the high-frequency transient eddies is examined using an idealized barotropic model. Although some limitations for practical applications are evident due to the simplicity of the model, we point out some important implications for storm track dynamics that have not been appreciated by preceding studies on storm tracks.

In the absence of damping other than biharmonic diffusion, on an \( f \) plane the transient eddy vorticity is almost conserved when the basic wind field possesses moderate inhomogeneity that resembles the typical NH winter stationary wave, whereas the streamfunction and the EKE are not; as the eddies are elongated meridionally in the jet minimum, the amplitude of the streamfunction and the EKE decrease. This is demonstrated by a simple scaling argument, a WKBJ solution, and a numerical solution. As can be seen in the WKBJ solution, as long as \( \omega^2 \gg |4U_U y|, k(x)(|\psi|) \) increases (decreases) as \( U \) decreases. In our calculation, this result holds even when \( |4U_U y| \) is \( O(1) \). In observations, the meridional potential vorticity gradient \( Q_y \) would replace \(-U_U y\). From observations, we estimate that in the upper troposphere \( 4UQ_y \approx 5 \times 10^{-9} \) s\(^{-2}\). This value is very close to the value of \( \omega^2 \) when the wave’s period is one day. Clearly this period is too short to be considered as that of typical synoptic-scale eddies. However, considering that our model is highly idealized, perhaps the value given above should not be taken as a precise one. Therefore, we cautiously expect that our result, namely, the decrease of \( |\psi| \) in the jet minimum, would be valid for the high-frequency eddies in the atmosphere. We note that the second storm track appears with no “source” of any type of instability, but it manifests itself simply due to kinematic modulation. In this simple model, the decrease and increase of EKE in the jet exit and entrance, respectively, are also energetically consistent. In Fig. 13, we show the barotropic energy conversion, indicating waves lose (gain) energy barotropically in the jet exit (entrance) as one expects for a meridionally elongated eddy. Energetic analyses of bandpass data by R. Dole (1993, personal communication) also exhibit the dipole structure in the vicinity of the jet exit and jet entrance, particularly at the Pacific jet exit and Atlantic jet entrance. The source and sink of EKE might tempt one to interpret it as a result of some sort of instability, but once again it appears as a source and sink because EKE is a locally (and globally) nonconserved quantity, not because of any type of instability. Using an example of vertically propagating planetary waves, Plumb (1983) gives a fuller discussion on why caution is needed in interpreting conventional energetic diagnostics.

These results imply that it is possible for the localized storm tracks in the atmosphere to appear due to barotropic modulation of the baroclinic waves, in the absence of local instability. This is not to claim that the baroclinicity is unimportant for the storm tracks in the atmosphere but to suggest another possibility for the localization of the storm tracks. It is particularly attractive to think of the storm tracks as a result of modulation, either baroclinic or barotropic, of a coherently propagating upper-tropospheric disturbance; this mechanism does not require the atmospheric basic state to be absolutely unstable, or the unstable mode to propagate slowly, for the existence of the storm tracks. As Lin and Pierrehumbert (1993) discuss, the atmosphere is unlikely to be absolutely unstable, although it is near the boundary of absolute instability. Growth of baroclinic waves due to absolute instability can be thought as Type A cyclogenesis, whereas growth due to baroclinic modulation of a wave packet can be thought of as Type B cyclogenesis. In the atmosphere, it happens that the location of growth and decay of synoptic-scale waves due to baroclinic modulation coincides with that due to barotropic modulation. As shown in this paper, to the extent that one trusts the wave maker’s capability to represent baroclinic eddies, this effect can be separately examined, and it would be of interest to investigate the relative importance of the two kinds of modulation using simple models that include both effects separately.

Although we discuss in this paper that localized storm tracks can exist in the absence of absolute instability, we suppose that Type A cyclogenesis occurs in the atmosphere as well, because it is conceivable that occasionally an instantaneous flow can support absolute instability even when the time-mean flow cannot. It is also possible that these three different mechanisms, that is, Type A and Type B cyclogenesis and barotropic modulation, dominate at different times of the year, or at different locations. Understanding different mechanisms is essential for solving puzzles related to interannual variability (Blackmon et al. 1977) of storm tracks and other related phenomena.

In our experiments, as the stretching deformation rate of the basic state increases, dissipation of enstrophy due to scale-selective diffusion increases dramatically, especially in the jet entrance region. The result is that the first storm track, measured by streamfunc-
tion, EKE, and enstrophy, is simply terminated, and the second storm track downstream is very weak. For the reasons discussed in section 4b, while we are not quite satisfied, analysis of zonal phase speed and momentum flux suggests that the dissipation in the jet entrance region may be explained by linear critical-layer dynamics. However, we point out that the existence of a critical layer in the jet entrance depends on the structure of the eddy, among others, and therefore it is by no means a general result, although in the atmosphere the high-frequency eddies tend to be meridionally elongated as in our experiment. We also argue that the enhanced dissipation can be explained by a simple, kinematic stretching mechanism when the background vorticity gradient is small. Because the potential vorticity gradient in the upper troposphere is not small in general, it is of interest to what extent the latter mechanism can be relevant in the atmosphere. In the upper troposphere at least, where barotropic decay is known to occur, the termination of the storm track due to this type of enstrophy cascade is appealing. However, as shown in Chang and Orlanski (1993), surface friction can also play a crucial role in terminating a storm track.

The sensitivity of our solution to the strength of the basic-state stretching deformation rate is especially interesting, because low-frequency variability affects the structure of the quasi-stationary waves, and in turn the corresponding structure of the storm tracks (Lau 1988; Dole 1986). Changes in the local baroclinicity of the basic flow might be able to explain the sensitivity of the storm track structure for different quasi-stationary waves, but the sensitivity is also consistent with our solutions—namely, a stronger stretching deformation rate can terminate a storm track more sharply.

One of the limitations of our model is that the baroclinic wave is mimicked by implementing a wave maker. Although this idealization allowed us to examine the effect of barotropic modulation free from any baroclinic modulations or instability, it does not capture the correct dynamics behind the baroclinic waves. It would be useful to construct some sort of systematic simple models to investigate the effects of realistic dynamics on our results.

One obvious extension of this work is to study nonlinear experiments. One would expect that the basic flow will be modified by the waves, and the consequence of that effect is unclear, although the study of Shutt (1983) shows that the basic-state deformation field can be maintained by the transient eddies in nonlinear calculations. In the nonlinear experiment, a wave maker designed to produce a wave packet or wave pulse is a desirable choice, because the solution will be different depending on whether the waves are of the form of wave packets or monochromatic waves, unlike the linear solution. Another useful extension of this work is to examine the isentropic potential vorticity (IPV) in the upper troposphere (350- or 330-K surfaces) and to compare these with more conventional measures of storm tracks using observed data. In the atmosphere, it is the IPV that is directly analogous to the vorticity in the two-dimensional, incompressible, homogeneous fluid that we considered in this study. We hope that this analysis will be able to convincingly tell us the relevance of our results for the atmosphere.

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