The Role of Eddy Diffusivity on a Poleward Jet Shift

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ABSTRACT

The authors use a quasigeostrophic (QG) two-layer model to examine how eddies modify the meridional asymmetry of a zonal jet. The initial asymmetry is introduced in the model’s “radiative equilibrium state” and is intended to mimic a radiatively forced poleward jet shift simulated by climate models. The calculations show that the initial “poleward” jet shift in the two-layer model is amplified by eddy potential vorticity fluxes. This eddy-accentuation effect is greater as the baroclinicity of the equilibrium state is reduced, suggesting that seasonal variations in baroclinicity may help explain observed and modeled jet-shift sensitivity to season. The eddy-accentuated jet shift from the corresponding radiative equilibrium state is more clearly visible in the slowly varying, eddy-free reference state of Nakamura and Zhu. This reference state formally responds only to nonadvective, nonconservative processes, but ultimately arises from the advective eddy fluxes. The implication is that fast eddies are capable of driving a slowly varying jet shift, which may be balanced by non-conservative processes such as radiative heating/cooling.

1. Introduction

Satellite observations and reanalysis products show evidence of poleward shifts of the mean latitude of the extratropical storm tracks (McCabe et al. 2001; Fyfe 2003; Bender et al. 2012) and jets (Lee and Feldstein 2013). Similar jet shifts have also been identified in the ensemble of twenty-first-century climate simulations (Yin 2005; Tsushima et al. 2006; Meehl et al. 2007; Barnes and Polvani 2013). Kushner et al. (2001) showed that CO2 increases can drive poleward jet shifts. Polvani et al. (2011) showed that stratospheric ozone depletion can also contribute to the poleward jet shift in the Southern Hemisphere (SH). The poleward shift of storm tracks accompanies similar trends in the surface westerlies and precipitations. Therefore it is of great interest to better understand the mechanisms of the jet- and storm-track shifts.

Grise and Polvani (2014a) show with the 4×CO2 experiments of CMIP5 archives that direct radiative forcing from the CO2 causes a poleward jet shift but that this shift accounts for a minor fraction of the total jet shift simulated by the models. Instead, the majority of the jet shift is associated with models’ sea surface temperature (SST) change. Using a reanalysis dataset, Lee and Feldstein (2013) found that the SH poleward jet-shift pattern, associated with higher global-mean temperature, occurs over 7–11-day time scales and that the decadal trend in the jet shift is realized through more frequent occurrence of these short-time-scale events. In addition, those poleward shift events are often preceded by tropical convection anomalies (Feldstein and Lee 2014). Since tropical convection is closely tied to the underlying SST, the SST-jet shift connection in Grise and Polvani (2014a) may also involve short-time-scale events. Consistent with this possibility, Wu et al. (2013) show that when CO2 is raised instantaneously, the atmosphere adjusts rapidly within a season.

How do these jet shifts occur over the short time scales? The most likely process is the synoptic-scale wave growth and decay, also known as the nonlinear baroclinic life cycle, first shown through numerical calculations of idealized baroclinic waves (Gall 1976; Simmons and Hoskins 1978, 1980). These studies
showed that unstable baroclinic waves initially grow, as is expected from the linear theory (Charney 1947; Eady 1949), but once they reach finite amplitude, the wave activity radiates equatorward and poleward, away from the wave source, culminating in zero wave amplitude and an altered mean state through irreversible mixing of potential vorticity (PV). If the equatorward wave propagation is greater, the jet shifts poleward in the altered mean state. The opposite occurs if the poleward wave propagation dominates (Thorncroft et al. 1993; Akahori and Yoden 1997; Martius et al. 2007). Because $\beta$ is greater on the equatorward side, waves preferably propagate toward the equator. The existing theories for the poleward jet shifts—faster eddy phase speeds (Chen and Held 2007; Lu et al. 2008), a rise of tropopause (Lorenz and DeWeaver 2007), an amplification of baroclinic waves triggered by tropical convection and subsequent enhancement of equatorward wave propagation (Park and Lee 2013; Feldstein and Lee 2014), or a shift in the reflecting latitude (Lorenz 2014)—all involve wave dynamics and/or irreversible eddy mixing in the jet’s equatorward side. This physical picture is underscored in Nie et al. (2014).

Given the fact that the midlatitude jets are driven by eddies, the aforementioned role of the fast, synoptic-scale waves on the jet shift is perhaps to be expected. On the other hand, since the waves tend to grow where meridional temperature gradient is enhanced, non-advective (nonconservative) and slow diabatic processes, such as cloud-radiative forcing, have been identified as being important players in the jet shifts (Ceppi et al. 2012, 2014; Li et al. 2015). However, Grise and Polvani (2014b) showed that at least in some models such cloud-radiation forcing is in fact induced by dynamics.

The goal of this study is to illustrate 1) how irreversible eddy mixing amplifies a jet shift caused by radiative forcing and 2) how irreversible eddy mixing influences a slowly varying background state. In particular, this influence may be manifested by nonconservative processes such as cloud-radiative heating/cooling in climate models. For these purposes, we employ a two-layer QG model where we mimic the eddy-free response to anthropogenic greenhouse gas (GHG) forcing by perturbing the model’s radiative equilibrium state such that it introduces a meridional asymmetry (Grise and Polvani 2014a). A similar line of investigation has been performed with climate models (Butler et al. 2010; Lu et al. 2014). In these studies, jet response to tropical heating was investigated where the tropical heating is derived from climate models’ time-mean response to GHG forcing. Since midlatitude eddies influence tropical heating (Kim and Lee 2001; Haqq-Misra et al. 2011), the imposed tropical heating may in fact embody the effect of the eddies in the first place. More importantly, owing to the spherical geometry in those models, the tropical heating also causes Hadley circulation and subtropical jet to change. The two-layer QG model is highly idealized, but its channel geometry allows one to examine midlatitude dynamics in isolation from potential influence by tropical circulations. In addition, the diagnostics that we develop and employ for goal 2 is simpler.

For the purpose of addressing goal 2, we obtain non-advective, slowly varying components of the atmosphere. We construct an “eddy free” reference state for the two-layer QG model, following the method of Nakamura and Zhu (2010, hereafter NZ10). Through this diagnostic, we show that the nonadvective effect of eddy mixing shifts the jet farther “poleward” than the initially perturbed radiative equilibrium state [consistent with Grise and Polvani (2014a)], and that the non-advective, slowly varying state is in fact driven by the fast eddies [consistent with Grise and Polvani (2014b)].

This paper is organized as follows. We first provide a brief description of the two-layer QG model and a theoretical framework for diagnosing a slow, non-advective evolution of the mean state in the context of the model. We then examine whether and how non-advective states differ from their respective radiative equilibrium states with or without meridional asymmetry. In addition, we conduct sensitivity tests to evaluate how the nonadvective state is affected by the domainwide baroclinicity.

### 2. Methodology

#### a. Model

Following Nakamura and Wang (2013, hereafter NW13), we use the nondimensional equations for the two-layer QG model with an unequal-layer thickness on a beta plane:

\[
\frac{\partial q_1}{\partial t} + J(\psi_1, q_1) = -\sigma^{-1}(\psi_2 - \psi_1 + \psi_R) - \kappa \nabla^2 \psi_1, \\
\frac{\partial q_2}{\partial t} + J(\psi_2, q_2) = \sigma^{-1}(\psi_1 + \psi_R - \gamma^{-1} \nabla^2 \psi_2) - \kappa \nabla^2 \psi_2, 
\]

(1)

where the quasigeostrophic PV is

\[
q_1 = \beta y + \nabla^2 \psi_1 + \frac{\psi_2 - \psi_1}{2(2-\delta)}, \\
q_2 = \beta y + \nabla^2 \psi_2 - \frac{\psi_2 - \psi_1}{2\delta}. 
\]

(2)
The subscripts 1 and 2 refer to the upper and lower layers, respectively. The nondimensional $\beta$ measures the ratio of planetary vorticity gradient to vertical shear contribution. The parameter $\delta$ denotes nondimensional thickness of the lower layer at rest (the upper-layer thickness is $2 - \delta$ and $\delta = 1$ corresponds to equal thicknesses). As discussed in NW13, this parameter controls the vertical structure of baroclinic instability and the associated criticality of the mean state. The flow becomes subcritical when $\beta\delta > 1$. The velocity field is determined by the relation $(u_i, v_i) = (-\partial \psi_i/\partial y, \partial \psi_i/\partial x)$. The time is nondimensionalized by $L_d/U$, where horizontal length scale is $L_d = 750$ km and velocity scale is $U = 45$ m s$^{-1}$. Hyperviscosity is included in both layers to remove enstrophy at small scales. Ekman damping with a damping time scale $\gamma$ of 0.4 day is included in the lower layer only, and thermal relaxation of the upper-layer zonal-mean flow set to a prescribed “radiative equilibrium state” $U_i = -\partial \psi_i/\partial y$ and the lower layer set to zero wind are adopted with a relaxation time scale of 30 days.

Equation (1) is solved numerically with Fourier spectral decomposition in the zonal direction and with sine-function decomposition in the meridional direction. As shown in NW13, a shallower lower layer can yield a more realistic flow regime and a more monotonic mean potential vorticity gradient in both layers, which is favored by the forthcoming Lagrangian diagnostics; therefore, in this study we choose a layer thickness ratio of $\delta = 0.25$. The key findings in this study have been tested with other values (e.g., standard equal-layer thickness) and have been confirmed to be insensitive to the particular value of choice. The nondimensional channel length and width are set to $L_z = 20\pi$ and $L_y = 10\pi$, respectively. The width chosen is sufficiently large so that eddy amplitude is negligible near the walls. A sponge layer is added at both northern and southern boundaries to avoid reflecting waves. For the parameters chosen here, the value of the corresponding non-dimensional $\beta$ is 0.2.

To initialize the numerical integration, a localized, small-amplitude (maximum amplitude of 0.01) perturbation is added to the upper-layer eddy PV field. Since such a perturbation excites all wavenumbers in spectral space, the fastest-growing normal mode naturally emerges.

b. Eddy-free reference state

On short time scales the dynamics of large-scale eddies are dictated by advection of PV, the second term on the left-hand sides of (1). However, in the long-term response of the eddy-driven jet to climate forcing, the role of nonconservative processes [the right-hand-side terms of (1)] is not negligible. To quantify the latter, it is useful to diagnose the part of the climate state that responds only to nonadvective processes. Such a “slowly varying reference state” may be defined through PV as a function of equivalent latitude $Y$ (NZ10; Nakamura and Solomon 2010, 2011; Methven and Berrisford 2015) instead of Eulerian latitude $y$. Given a monotonic PV gradient, equivalent latitude is the latitude that encloses the same area (for the QG dynamics) or layer mass (for isentropic dynamics) on its poleward side as the corresponding wavy contour of PV (Butchart and Remsberg 1986; Allen and Nakamura 2003; Nakamura and Solomon 2011). In a QG model on the $\beta$ plane, PV in equivalent latitude does not change in time when the dynamics are purely advective, because the advecting horizontal wind preserves the area enclosed by a PV contour. When the dynamics involves diffusivity of PV, radiative forcing, and Ekman damping (in the lower layer), PV on the equivalent latitude evolves solely because of the corresponding nonconservative terms on the right-hand side of (1):

$$\frac{\partial}{\partial t} Q_i(Y, t) = \frac{\partial}{\partial Y} \left( K_i \frac{\partial Q_i}{\partial Y} \right) + R_i + F_i; \quad i = 1, 2,$$  (3)

where $Q_i$ is PV with respect to equivalent latitude in each layer and may be thought of as the zonally symmetric distribution of PV that arises after one “zonalizes” the wavy PV contours in each layer without changing the areas that they demarcate. On the right-hand side of (3), $K_i(Y, t)$ is effective diffusivity of $Q_i$ (Nakamura 1996), which combines the effects of hyperdiffusion in (1) and large-scale stirring; $R_i(Y, t)$ is radiative effects on $Q_i$; and $F_i$ represents friction, applied only in the lower layer [see (28) and appendix of NZ10 for the derivation of (3) above].

NZ10 also introduces an eddy-free reference-state flow (hereafter $U_{REF}$) by inverting PV gradient in equivalent latitude. In the context of the QG two-layer model above,

$$\frac{\partial Q_1}{\partial Y} = \beta - \frac{\partial^2 U_{REF1}}{\partial Y^2} + \frac{U_{REF1} - U_{REF2}}{2(2 - \delta)};$$  (4a)

$$\frac{\partial Q_2}{\partial Y} = \beta - \frac{\partial^2 U_{REF2}}{\partial Y^2} - \frac{U_{REF1} - U_{REF2}}{28}.$$  (4b)

Here, $[(2 - \delta) \times (4a) + \delta \times (4b)]/2$ relates the barotropic components of PV gradient and $U_{REF}$

$$\left(\frac{2 - \delta}{2}\right) \frac{\partial Q_1}{\partial Y} + \frac{\delta}{2} \frac{\partial Q_2}{\partial Y} = \beta - \frac{\partial^2}{\partial Y^2} \left[\left(\frac{2 - \delta}{2}\right) U_{REF1} + \frac{\delta}{2} U_{REF2}\right],$$  (5)
whereas (4a) minus (4b) yields the baroclinic relation
\[
\frac{\partial Q_1}{\partial Y} - \frac{\partial Q_2}{\partial Y} = \frac{1}{2} \left( \frac{1}{2 - \delta} - \frac{1}{\delta} \right) (U_{\text{REF}1} - U_{\text{REF}2})
\]
\[\quad - \frac{\partial^2}{\partial Y^2} (U_{\text{REF}1} - U_{\text{REF}2}). \tag{6}\]

Equations (5) and (6) may be solved numerically for $U_{\text{REF}1}$ and $U_{\text{REF}2}$ with prescribed boundary values of $U_{\text{REF}}$.

In the limit of the free-decaying barotropic fluid of NZ10, the relationship between $U_{\text{REF}}$ and its equilibrium state is simple: $U_{\text{REF}} = U + A$, where $A$ is finite-amplitude wave activity. In a two-layer baroclinic fluid, however, there is the residual circulation that relates the circulation of one layer to that of the other layer.

Therefore, to solve for $U_{\text{REF}}$ it is necessary to have an appropriate lower boundary condition.

Since $Q_i$ responds only to nonadvective processes, so does $U_{\text{REF}}$. If the total flow is entirely devoid of eddy (i.e., zonally symmetric), then the first term on the right-hand side of (3) is negligible and both $Q_i$ and $U_{\text{REF}i}$ will asymptotically approach their radiative equilibrium profiles at large $t$ ($U_{\text{REF}2} = U_0$ and $U_{\text{REF}1} = U_0 + U_e$, where $U_0$ is the global-average $u$ in the lower layer).

Thus, the deviation of $U_{\text{REF}}$ from $U_e$ may be interpreted as being a result of the irreversible PV mixing (non-advective) by the eddies, whereas the deviation of the zonal-mean zonal wind from $U_e$ mostly reflects the total effect of the eddies including advective and non-advective processes.

![FIG. 1. The radiative equilibrium state $U_e$ (m s$^{-1}$) and the associated PV gradient $dq_e/\partial Y$ (10$^{-11}$ m$^{-1}$ s$^{-1}$) for (a),(b) the CTRL experiment and (c),(d) the SKEW experiment. One unit in the horizontal axis corresponds to one radius of deformation $L_d = 750$ km.](image)
Throughout this study, as conventional in the Northern Hemisphere, we refer the direction of $y \to -\infty$ as “equatorward.” We use a meridionally symmetric radiative equilibrium state $U_e$ for a control run (referred to herein as CTRL):

$$ U_e = \Delta U + \exp \left[ \frac{(y - y_c)^2}{2\alpha^2} \right], \quad (7) $$

where $\Delta U$ is the mean vertical shear, which is set to zero in CTRL, $y_c$ is the meridional midpoint of the channel, and $\alpha = 1.7$ scales the jet’s width. Using (7), the CTRL $U_e$ profile is shown in Fig. 1a. Figure 4 of Grise and Polvani (2014a) shows that the radiatively induced jet shift is very small compared to the total jet shift, especially during DJF when the jet is mostly eddy driven, suggesting that the direct radiative effect by GHGs can be modeled as a small perturbation to our CTRL radiative equilibrium state. Aiming to mimic a jet shift caused by a direct radiative effect of an increased GHG loading, we add a perturbation on the meridional symmetry of $U_e$ in CTRL such that the shape is meridionally skewed with its maximum being shifted poleward. At the same time, we also take into account the effect of spherical geometry on the PV gradient. We refer this meridionally skewed $U_e$ profile as SKEW (see the
appendix for a detailed description). Specifically, comparing with CTRL (Figs. 1a and 1b), the peak of $U_1$ in SKEW (Fig. 1c) is shifted poleward by 0.8 units in $y$. The asymmetry in $U_1$ for the SKEW experiment results in a maximum $\frac{d}{dy}(\tilde{q}_1)$ at 0.4 units in $y$. This is introduced to mimic the fact that $\beta$ is greater on the equatorward side of the jet.

3. Numerical model calculations

a. Reference experiments

Both CTRL and SKEW are integrated for 10,000 days and the outputs from the last 8000 days are used for analysis. The resulting 8000-day-average statistics are presented in Fig. 2 (see also Fig. 5). The eddy-free reference state is solved numerically using (6), assuming no-slip boundary condition, $U_{REF} = 0$, to represent the effects of surface friction (Nakamura and Solomon 2010). For clarity, we refer to $U_{REF}$ as $U_{REF}$.

Figure 2 shows that in SKEW the upper-layer mean zonal wind $U_1$ is displaced poleward of the initial radiative equilibrium state. Similarly, the peak of the corresponding PV gradient $d(\tilde{q}_1)/dy$ is also displaced farther poleward than that of the radiative equilibrium state. However, the maxima of $U_1$ and $d(\tilde{q}_1)/dy$ in Fig. 2 are less sharp than their radiative equilibrium counterparts.

In comparison, the eddy-free reference state (Fig. 2) $U_{REF}$ and $\frac{d}{dy}(\tilde{q}_1)$ exhibit sharper and well-defined
maxima. More importantly, the jet shift in \( U_{REF} \) is 1.48 units of \( y \), which is 1.85 times the peak shift of \( U_e \) (0.8 units of \( y \)). The difference between \( U_{REF} \) and \( U_e \) represents nonadvective, nonconservative, “diabatic” effects. However, as was discussed earlier, this slowly varying change in diabatic effects in fact arises from eddy activity, because in the absence of eddies the solution to the system is \( U_e \) in the upper layer and zero wind in the lower layer.

\subsection*{b. Transition period}

In SKEW, most of the poleward shift of \( U_{REF} \) occurs within a relatively short time period. By focusing on this transition period, we examine the correspondence between changes in \( U_{REF} \) and strength of eddy fluxes. To filter out the internal variability, we perform a 50-member ensemble of SKEW with slightly different initial perturbations at rest. The ensemble is constructed by adding random perturbations to the initial perturbation. Specifically, the perturbation has an amplitude of 0.01\( \sigma \), where \( \sigma \) is a random number between 0 and 1, and the perturbation is introduced to the spectral coefficient of zonal wavenumber 3 and meridional wavenumber 5. Since the amplitude of this additional perturbation is on the same order of magnitude as that of the initial perturbation, the perturbations rapidly interact with each other. Therefore, once the flow reaches finite amplitude, the resulting flow field diverges from each other and the jets evolve with different internal variability. Upon averaging 50 ensemble members, this internal variability is removed.

Strong eddy fluxes lead to large finite-amplitude wave activity (FAWA), which induces changes in \( U_{REF} \) indirectly via irreversible mixing. This can be understood by the large FAWA limit of the barotropic FAWA equation as in NZ10 (24a):

\[
K_1 \frac{\partial Q_1}{\partial Y} = -u'_1 q'_1 - \frac{\partial A_1}{\partial t} + \Delta \Sigma_1, \tag{8}
\]

where \( A_1 \) is FAWA defined by NZ10, \( K_1 \) is the effective diffusivity, \( K_1 \frac{\partial Q_1}{\partial Y} \) is the diffusive flux of PV, and \( \Delta \Sigma \) is the radiative damping. The eddy PV flux is predominantly balanced by FAWA tendency and diffusive flux (radiative damping contribution is relative small). The eddy PV fluxes first enhance FAWA, which then decays through enstrophy dissipation associated with wave breaking and eddy mixing. The nonconservative decay of FAWA (i.e., the diffusive flux of PV), which is driven by the advective eddy fluxes in the first place, can modify \( K_1 \) as in (3) or equivalently \( \frac{\partial Q_1}{\partial Y} \), which then modifies \( U_{REF} \) through the inversion relation in (4).
As an indirect driver of $U_{\text{REF}}$, advective eddy fluxes of PV are connected to the eddy momentum flux divergence and eddy thickness (heat) flux through the Taylor identity:

$$\overline{u_1' q_2'} = -\frac{\partial}{\partial y} \overline{u_1' u_1'} + \frac{1}{2 \times (2 - \delta)} \overline{v_1' (\psi_2' - \psi_1')} \overline{\bar{u}'}, \quad (9a)$$

$$\overline{v_2' q_2'} = \frac{\partial}{\partial y} \overline{u_2' v_2'} - \frac{1}{2 \delta} \overline{v_2' (\psi_2' - \psi_1')} \overline{\bar{v}'}. \quad (9b)$$

Based on the ensemble, we construct a composite relative to the time in each simulation when the peak $Y$ of $U_{\text{REF}}$ first shift poleward of $Y = 1.47$, the latitude of statistically steady $U_{\text{REF}}$ (Fig. 2c). This reference time is denoted as lag 0 day, and the composite results are...
shown in Fig. 3. It can be seen that initially $U_{\text{REF}}$ shifts slightly equatorward between days $-40$ and $-25$. The brief, initial equatorward shift of $U_{\text{REF}}$ reflects the poleward increase of eddy heat flux. Between days $-25$ and $-20$, a major poleward shift of $U_{\text{REF}}$ occurs abruptly. The subsequent posttransition jet latitude is statistically different from the pretransition jet latitude (defined as an average between days $-100$ and $-50$) at the 95% confidence level in a two-tailed Student’s $t$ test.

As a precursor, the change in $U_{\text{REF}}$ coincides with the development of a large eddy heat flux. This eddy heat flux leads to an increasing FAWA, facilitating equatorward PV mixings. During this process, the eddy heat flux dominates (over the eddy momentum flux convergence) the contribution to the eddy PV flux. The corresponding strong diffusive flux drives $\partial Q_1/\partial Y$ via the process expressed by (3) and leads to a sink of FAWA and a shift in $U_{\text{REF}}$. Between days $-10$ and $0$, initiated by a few life cycles with strong eddy heat fluxes, the enhanced mixing of $Q_1$ leads to a stronger $\partial Q_1/\partial Y$, which subsequently sharpens $U_{\text{REF}}$.

In Fig. 4, the PV snapshots illustrate the evolution of flow patterns of wave breakings that occur during the jet shift. On days $-50$ and $-40$, the perturbation has small amplitude and the flow is governed by linear dynamics. FAWA and eddy fluxes are both growing during time period. By day $-28$, the flow develops into finite amplitude with appreciable meridional tilt. On day $-23$, when $U_{\text{REF}}$ shifts poleward abruptly, the upper-layer PV field is characterized by the development of a strong cold front and slightly weaker warm front, which is qualitatively consistent with the typical anticyclonic (LC1) wave breaking (Thorncroft et al. 1993). This LC1-type wave breaking leads to irreversible mixings of PV. This is consistent with the life cycle simulations using an idealized AGCM in Solomon et al. (2012), who show that LC1 wave breaking results in significant mixing on the equatorward flank of the jet.

c. Sensitivity experiments

We next investigate the sensitivity of our main result to domainwide baroclinicity in a two-layer QG model. In nature, the change of domainwide baroclinicity could be caused by a seasonal cycle or a reduced equator-to-pole temperature gradient that tends to occur in warm climates. To investigate the extent to which the eddy-accentuated jet shifts is influenced by domainwide
baroclinicity, we conduct sensitivity experiments by systematically varying the vertical wind shear $\Delta U$ in $U_e$. As $\Delta U$ increases, the flow becomes more supercritical to baroclinic instability.

Figure 5 shows that as the baroclinicity is increased, stronger baroclinicity results in lesser poleward jet shift. Under the strongest baroclinicity that we consider ($\Delta U = 7.5 \text{ m s}^{-1}$ in the cyan curves), the shift of $U_{\text{REF}}$ is only 1.4 times that of $U_e$. One possibility is that as the baroclinicity is increased, the upper-layer wind also strengthens, hence suppressing irreversible PV mixing. However, as Fig. 6 shows, the difference between the equilibrium PV gradient and the nonadvective state’s PV gradient for the strong baroclinicity case is even greater (in their magnitude) than that for the weak baroclinicity case. Because nonadvective states are primarily modified by eddy diffusive flux, these results indicate that the muted jet shift in the strong baroclinicity case is not because of a suppressed PV mixing.

We hypothesize that the reason for this sensitivity is related to the changes in the meridional structure of diffusive flux of PV. Figure 7 shows that the meridional distribution of diffusive flux of PV is more uniform in the strong baroclinicity case than in the weak baroclinicity case. In the latter, the flux shows a notable peak on the equatorward flank of the jet. As the domainwide baroclinicity increases, wave amplitude and meridional parcel displacement tend to increase as well. As a result, waves that radiate from the jet center can break closer to the jet axis (before they reach their linear critical latitudes) and stir PV over a broader meridional extent. Under this circumstance, wave breaking and the irreversible mixing would start to occur closer to the initial jet center and the meridional extent of mixing would be greater. According to the idea presented in sections 3a and 3b that an enhanced wave breaking on the equatorward side has an important impact on the diffusive flux and on the poleward jet shift, such a broad wave...
breaking would lead to a less concentrated diffusive PV flux on the equatorward flank of the jet—hence, a less pronounced jet shift.

4. Conclusions and discussion

In this study, we quantified the effect of eddy mixing on a jet shift caused by an initial radiative forcing. Our results indicate that the eddy effect is to accentuate the initial jet shift. The eddy-accentuated permanent jet shift requires only a few life cycles, largely associated with LC1-type wave breaking. Our results are consistent with Nie et al. (2014, 2016), who show that irreversible potential vorticity mixing plays a critical role in driving and maintaining the jet shift. In particular, Nie et al. (2016) demonstrate that when lower-tropospheric thermal forcing is changed, the barotropic eddy response (represented by a diffusive eddy PV flux) dominates the total atmospheric response. Our two-layer QG results, with a deep upper layer, are consistent with their findings. Furthermore, by constructing a slowly varying reference state following the method of NZ10, we also evaluate how the driving by fast eddies is manifested in the form of slowly varying state, which only responds to nonadvective processes.

There are two implications of the result for the current jet-shift debate. First, our result provides a plausible explanation for the finding by Grise and Polvani (2014a) that eddies can amplify a GHG-driven jet shift. Second, if one does not have the information about the radiative equilibrium state, eddy-free diagnostics alone may seem to attribute the cause of the jet shift to nonconservative diabatic processes such as convective and radiative heating/cooling. However, with an explicitly prescribed radiative equilibrium state in a two-layer QG model, we demonstrate that the deviation of the slowly varying eddy-free state from the radiative equilibrium state arises predominantly from the eddy mixing.

Our calculations also indicate that the eddy effect on a jet shift is more pronounced as the baroclinicity of the initial state is weakened. Simpson et al. (2014) show that high-frequency eddies can drive a stronger poleward jet shift in SH summer than in winter (their Fig. 4). The formation of an ozone hole in spring/summer plays a role in this seasonality. However, because the jet shift is sensitive to baroclinicity, as shown here, it may be that the weaker baroclinicity during the summer can also help explain the earlier finding that the SH jet shift is particularly pronounced during summer (Barnes and Polvani 2013; Bracegirdle et al. 2013; Simpson et al. 2014). This possibility is expected to be tested more readily with the recovery of the Antarctic ozone hole (Solomon et al. 2016).
In addition, we find that weak baroclinicity allows for a smaller range of variability in the jet shift. Figure 8 shows meridional profiles of the standard deviation of \( U_{\text{REF}} \) for four cases of differing baroclinicity. The standard deviation is obtained from an 8000-day integration for each case. There are three notable features. First, the standard deviation increases as baroclinicity is increased, and this is true at all latitudes. Second, for each case, the standard deviation is a minimum at the jet center and a maximum at jet flanks. The distance between the minimum and the maximum standard deviation increases with baroclinicity. These features indicate that jet variability is greater both in amplitude and meridional extent as baroclinicity is increased. This is consistent with the fact that the PV gradient is broader and flatter in the strong baroclinic case (Fig. 5). This result is also consistent with a previous modeling work (Son et al. 2010) that the uncertainty range of jet shift is relatively small during summer.

Since the diffusive flux of PV drives changes in \( U_{\text{REF}} \) through the modification of \( K_1 {\partial Q_1}/ {\partial Y} \) in the two-layer QG model, the slowly varying, nonadvective (irreversible) component of the solution is ultimately driven by the eddies. In the atmosphere the force balance for the large-scale circulation requires that the circulation and the corresponding temperature (more generally density) fields are, to leading order, in thermal wind balance. Therefore, the temperature field is expected to respond to a diffusive PV flux and the generation of \( U_{\text{REF}} \). Such thermodynamic adjustment may be realized through vertical motion that can then influence static stability, convection, clouds, and radiation. Evidence of dynamically induced cloud and radiation was pointed out by Grise and Polvani (2014b) and was also shown in the context of the Madden–Julian oscillation (Lee and Yoo 2014).

The FAWA framework has recently been adopted to study the circulation response to climate change (Chen et al. 2013; Sun et al. 2013; Lu et al. 2014). However, the previous studies do not examine the slowly varying nonadvective state, and it is uncertain as to how an eddy-free reference state behaves in global reanalysis and climate models. This work therefore serves as a step toward a better understanding of the effect of diffusive eddy flux in a changing climate. The same diagnostics can be applied to global reanalysis and climate models (Nakamura and Solomon 2010, 2011) to evaluate the role of eddy induced diffusive PV fluxes in jet variability. Because the PV gradient structure fundamentally determines the eddy properties, which in turn feedback on the PV gradient structure itself, such a future work also needs to explore the PV gradient structure and its sensitivity to external forcing in a changing climate.

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APPENDIX

**SK EW Jet Profile**

In SKEW \( U_e \) was obtained by manipulating a skew Gaussian profile:

\[
 f(y) = \Delta U + \exp \left( -\frac{(y - y_c)^2}{2\sigma^2w^2} \right) \left\{ 1 + \text{erf} \left( \frac{\alpha(y - \eta)}{w\sqrt{2}} \right) \right\},
\]

(A1)

where \( \Delta U \) is the mean vertical shear which is set to zero in CTRL, \( y_c \) is the meridional midpoint of the channel, \( \sigma = 1.3 \) scales the jet width, \( \text{erf} \) denotes the error function, \( \eta = 2.05 \) locates the peak of the skewed jet, and \( \alpha = -1.7 \) is the skewness parameter.

In the atmosphere, the spherical effect makes PV gradient greater on the equatorward side; hence, waves propagate more readily equatorward. In our channel model, this effect on \( \beta \) is introduced by manipulating the \( U_c \) profile. The analytical formula \( f(y) \) in (A1) however, comes with the peak of the PV gradient \( dq/dy \) at the poleward side of the jet. The analytical skew Gaussian profile \( f(y) \) in (A1) is therefore further manipulated such that the peak of PV gradient is greater on the equatorward of the \( U_c \) peak. We define a piecewise continuous weighting function across the jet. Specifically, we follow three steps: 1) To switch the peak of the PV gradient to the equatorward, the \( f(y) \) profile in (A1) is mirror rotated with respect to its maximum. 2) An unintended consequence of the previous step is a production of a negative PV gradient at the equatorward flank of a rotated \( f(y) \), which is undesirable for the equivalent-latitude calculations where a monotonic PV distribution is required. We thus increase the PV gradient over an interval, \( [y_1, y_2] \), which encloses the negative PV gradient, by reducing the local curvature of \( f(y) \) at equatorward flank. The \( f(y) \) profile within \( y_1 \) and \( y_2 \) takes the form of \( 0.7f(y) + 0.3f_{\text{LINEAR}}(y) \), where the formula \( f_{\text{LINEAR}}(y) = f(y_1) + y \times [f(y_2) - f(y_1)]/(y_2 - y_1) \) is used for the linear interpolation. 3) A five-point moving-average
smoothing is applied to remove the discontinuities introduced by the previous step.

REFERENCES


